Kenilworth Public Schools

Curriculum Guide

Content Area: AP Calculus AB
Grade: 11-12
BOE Approved: 10/15/2013

Revision Date: June 2022
Submitted by: Lynne Smith
BOE Revision Approved: 8/8/22

AP Calculus AB Scope and Sequence

Unit 1- Functions, Graphs, and Limits	Unit 2- Differentiation	Unit 3- Integration	Unit 4- Transcendental Functions and Differential Equations	Unit 5- Take the AP Calculus AB Practice Exam	Unit 6- Final Exam
Weeks 1-4	Weeks 5-13	Weeks 14-20	Weeks 21-28	Weeks 28-33	Weeks 34-38
Unit Description: Find characteristics of functions and their graphs both graphically and analytically. Understand the concept of a limit. Find limits, or determine that they fail to exist, both analytically and when given a graph. Understand one-sided limits and limits at infinity (end behavior).	*Unit Description:* Understand the limit definition of derivative. Find derivatives analytically, using all rules. Use derivatives to find critical numbers, extrema, concavity, and points of inflection. Understand and use Rolle's Theorem and the Mean Value Theorem.	*Unit Description:* Understand the concept of antiderivatives and find indefinite integrals analytically. Understand the relationship of the area problem and calculus, estimate integrals using Riemann Sums, and find integrals using the Fundamental Theorem of Calculus. Find areas and volumes bounded by curves.	*Unit Description:* Understand the calculus definition of the natural logarithmic function. Find derivatives and integrals of the natural logarithmic function and exponential functions. Find derivatives of inverse functions. Use slope fields and solve differential equations.	*Unit Description:* Administer the College Board supplied AP Calculus AB Practice Exam, in accordance with the same time parameters. Provide the students with their graded exams, and an analysis of the class' performance, and of the class' performance relative to last year's actual AP Calculus AB exam total performance results. Identify areas where additional practice is needed. Utilize both individual and group guided practice to improve skills as needed.	*Unit Description:* Project-based final exam.
Unit Targets: • Functions and Graphs 1. Families of	*Unit Targets:* • Definition of and	*Unit Targets:* • Definition and	*Unit Targets:* • Understand the	*Unit Targets:* • Students will take a	*Unit Targets:* • Students will complete a

Functions 2. Domain and Range 3. Roots and Intercepts 4. Composite Functions 5. Linear Models 6. Interpret slope as a ratio or as a rate of change • Limits and their Properties 1. Estimate limits using a numerical or graphical approach 2. Limits that fail to exist 3. Formal definition of a limit 4. Properties of limits 5. Evaluate limits analytically 6. Squeeze Theorem • Continuity and one-sided limits 1. Continuity 2. One-sided limits 3. Intermediate Value Theorem 4. Infinite limits; using graphing calculators to view asymptotic	properties of derivatives 1. The tangent line problem; the limit definition of derivative 2. Understand the relationship between differentiability and continuity 3. When f' is underfined 4. Basic differentiation rules 5. Derivatives of trigonometric functions 6. Product and Quotient Rules 7. Higher-Order derivatives 8. Chain Rule 9. Implicit differentiation • Applications of differentiation 1. Related rates 2. Extrema on an interval; the Extreme Value Theorem 3. Relative extrema; critical numbers 4. Rolle's Theorem	properties of integrals 1. Antiderivatives and Indefinite Integration 2. Area 3. Riemann Sums and Definite Integrals 4. The Fundamental Theorem of Calculus 5. The Mean Value Theorem for Integrals 6. Average value of a function on an interval 7. The Second Fundamental Theorem of Calculus 8. Integration by Substitution; change of variables 9. Numerical Integration; the Trapezoidal Rule • Applications of Integration 1. Area of a region between two curves 2. Motion along a line	natural logarithmic and exponential functions in the context of calculus. Apply calculus operations to transcendental functions. 1. Develop and use properties of the natural logarithmic function 2. Understand the definition of the number "e" 3. Find derivatives of functions involving the natural logarithmic function 4. Use the definition of the natural logarithmic function to integrate a rational function 5. Integrate all six trig functions 6. Differentiate and integrate natural exponential functions 7. Differentiate and integrate exponential	practice AP exam • Lessons will be developed based upon necessary additional review/topics	project-based final exam after taking the AP exam.

behavior 5. Limits at infinity	5. Mean Value Theorem 6. Increasing and decreasing functions and The First Derivative Test 7. Concavity, points of inflection, and The Second Derivative Test 8. Optimization problems	3. Volume of a solid with known cross sections	functions that have bases other than "e" 8. Verify that one function is the inverse function of another function 9. Find the derivative of an inverse function • Use initial conditions to find particular solutions of differential equations 1. Use slope fields to approximate solutions of differential equations 2. Recognize and solve differential equations that can be solved by separation of variables. 3. Recognize limits that produce indeterminate forms 4. Apply L'Hopital's Rule to evaluate a limit		

AP Calculus AB Unit 1

Unit title: Functions, Graphs, and Limits

Unit summary: Various types of functions and their graphs are summarized to prepare students for calculus. A comprehensive study of limits is provided, including definitions of limits and one-sided limits, and procedures for finding limits both analytically and from graphs.

Primary interdisciplinary connections: Science, Technology, and Engineering

21st Century Themes: Problem Solving, Critical Thinking, Collaboration, Career Skills

Learning Targets

NJSLS Standards: 9.1.12.A.1, 9.1.12.F.2, F.BF.1, F.IF.1-2, F.IF.4-5. F.IF.7-9, A.SSE.1a-b, A.SSE.2-3, A.SSE.3a, F.TF.8

Content Statements:

1	Types of functions and graphs
2	Characteristics of functions and graphs
3	Estimate limits using numerical and graphical methods
4	Define limits and identify when limits fail to exist
5	Use properties of limits and evaluate limits
6	Understand and determine continuity, and identify types of point discontinuity
7	Use the Intermediate Value Theorem

Big Idea: Calculus is applied to mathematical models, or functions. Students must be familiar with the properties of functions, the composition of functions, the algebra of functions and the graphs of functions. Students must also understand the characteristics of various types of functions and their graphs, such as domain and range, odd and even, periodic, symmetry, zeros, intercepts, and increasing and decreasing. The idea of limits is essential for discovering and developing important ideas, definitions, theorems, and formulas. Students must have an intuitive understanding of limits and be able to compute various limits, including one-sided limits and limits at infinity. Students must be able to find limits analytically and from graphs.

Unit Essential Questions:

- Under what conditions does a limit exist, or fail to exist?
- How is the concept of limits extended to one-sided limits, limits at infinity, and infinite limits?
- How can numerical and graphical information be used to estimate limits?
- What are the mathematical properties of

Unit Enduring Understandings:

- The concept of a limit can be used to understand the behavior of functions.
- Continuity is a key property of functions that is defined using limits.

limits? • What types of continuities can exist, and in what types of functions?	

Unit Learning Targets
Students will…

- Express limits symbolically using correct notation.
- Interpret limits expressed symbolically
- Estimate the limits of functions
- Determine the limits of functions
- Deduce and interpret the behavior of functions using limits
- Analyze functions for intervals of continuity or points of discontinuity
- Determine the applicability of important calculus theorems using continuity

Evidence of Learning

Summative Assessment: Chapter Tests, AP Practice exam questions

Formative Assessments: Independent and group practice; classroom projects

- Independent Practice
- Group practice
- Classroom projects

Lesson Plans

Activities	*Timeframe*
• Explore slopes of secant lines: Using the graph of f(x)=$\sqrt{x}$, students are instructed to label the point (4,2) on the graph. Students then sketch the corresponding secant line and then calculate the slope between this point and (9,3), this point and (1,1), and this point and (4.41,2.1), in order to estimate the slope of the tangent line at the point (4,2). Students then write the slope of the secant line between (4,2) and (4+h, f(4+h)), and simplify the expression. This illustrates the "problem" with calculating the slope at a point because "h" cannot equal zero. A short time later, after students learn how to find limits analytically, we return to this problem and show that we can find the slope of the tangent line at (4,2) by calculating the limit as "h" approaches zero of $\sqrt{4+h}$ - $\sqrt{h}$ / h. We find it to be as we estimated, ¼. The same problem is referenced again when the limit	Weeks 1-4

definition of derivative is provided. Students with Disabilities, English Language Learners, and Gifted & Talented Students: Differentiating instruction is a flexible process that includes the planning and design of instruction, how that instruction is delivered, and how student progress is measured. Teachers recognize that students can learn in multiple ways. By providing appropriately challenging learning, teachers can maximize success for all students. Examples of Strategies and Practices that Support Students with Disabilities: • Use of visual and multisensory formats • Use of assisted technology • Use of prompts • Modification of content and student products • Testing accommodations • Authentic assessments Examples of Strategies and Practices that Support Gifted & Talented Students: • Adjusting the pace of lessons • Curriculum compacting • Inquiry-based instruction • Independent study • Higher-order thinking skills • Interest-based content • Student-driven instruction • Real-world problems and scenarios Examples of Strategies and Practices that Support English Language Learners: • Pre-teaching of vocabulary and concepts • Visual learning, including graphic organizers • Use of cognates to increase comprehension • Teacher modeling • Pairing students with beginning English language skills with students who have more advanced English language skills	

• Scaffolding •Word walls •Sentence frames •Think-pair-share •Cooperative learning groups •Teacher think-aloud	
Teacher Resources	*Teacher Note*
• Textbooks • Classroom set of graphing calculators • Graph paper • Textbook resources available for downloading and printing • AP College Board resources available for downloading and printing • Test Bank software	

AP Calculus AB Unit 2

Unit title: Differentiation

Unit summary: The concept of limits is used to define the slope at a point on a graph, or the derivative. Derivatives are estimated from tables of data and from graphs. Derivatives are determined and evaluated by using rules and properties. Derivatives are applied to solve a variety of real-life problems.

Primary interdisciplinary connections: Science, Technology, and Engineering

21st Century Themes: Problem Solving, Critical Thinking, Collaboration, Career Skills

Learning Targets

NJSLS Standards: 9.1.12.A.1, 9.1.12.F.2, F.IF.1-2, F.IF.4, F.IF.7, N.RN.2, A.SSE.1, A.SSE.3, A.APR.1

Content Statements:

1	The tangent line problem; the limit definition of derivative
2	Understand the relationship between differentiability and continuity.
3	When f' is undefined
4	Basic differentiation rules
5	Differentiation of trigonometric functions
6	Product and Quotient Rules
7	Higher order deriatives
8	The Chain Rule
9	Implicit Differentiation
10	Related rates
11	Extrema on an interval; the Extreme Value Theorem
12	Relative extrema, critical numbers
13	Rolle's Theorem and The Mean Value Theorem
14	Increasing and decreasing functions and the First Derivative Test
15	Concavity, points of inflection, and the Second Derivative Test
16	Optimization problems

Big Idea: Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyze the graph of a function, and solving problems involving rectilinear motion. Students will understand and apply the Mean Value Theorem, and will become familiar with a variety of real-world applications of derivatives.

Unit Essential Questions:	**Unit Enduring Understandings:**
• How is the average rate of change of a	• The derivative of a function is defined as the

function over a given interval defined? • How can limits be used to determine the instantaneous rate of change of a function? • What notations are used for derivatives? • What are the four modalities in which derivatives can be represented? • How are derivatives determined for sums, differences, products, and quotients of functions? • How are derivatives of composite functions determined? • How are derivatives of implicit functions determined? • How are higher order derivatives determined? • What characteristics of graphs can be determined from derivatives of functions? • What types of real-life problems can be determined with derivatives?	limit of a difference quotient and can be determined using a variety of strategies. • A function's derivative, which is itself a function, can be used to understand the behavior of the function. • The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. • The Mean Value Theorem connects the behavior of a differential function over an interval to the behavior of the derivative of that function at a particular point in the interval.

Unit Learning Targets
Students will…

- Identify the derivative of a function as the limit of a difference quotient
- Estimate derivatives
- Calculate derivatives
- Determine higher order derivatives
- Use derivatives to analyze properties of a function
- Recognize the connection between differentiability and continuity
- Interpret the meaning of a derivative within a problem
- Solve problems involving the slope of a tangent line
- Solve problems involving related rates, optimization, and rectilinear motion
- Verify solutions to differential equations
- Apply the Mean Value Theorem to describe the behavior of a function over an interval

Evidence of Learning

Summative Assessment: Chapter Tests, AP Practice exam questions

Formative Assessments:

- Independent Practice
- Group practice
- Classroom projects

Lesson Plans

Activities	*Timeframe*
• Explore reasoning from tabular data: Given that a particle moves along a horizontal line, with a positive velocity that is a function of t, provide students with a table of velocity values at selected times. Ask students to find the smallest number of times that the particle has a certain velocity for the t values in the table, and to justify their answer (Intermediate Value Theorem). Then ask students to find the smallest number of times that the acceleration of the particle could equal zero on the interval, and to justify their answer (Mean Value Theorem, because two velocities in the table will be the same). Ask the students to find the average acceleration between two t values that are in the table, showing all work. Finally, ask students to use a midpoint Riemann Sum with three intervals of equal length and values from the table to approximate the definite interval of v(t) dt from the minimum t value to the maximum t value in the table, and to explain the meaning of this definite interval in terms of the particle's motion (the distance travelled by the particle).	Weeks 5-13
Teacher Resources	*Teacher Note*
• Textbooks • Classroom set of graphing calculators • Graph paper • Textbook resources available for downloading and printing • AP College Board resources available for downloading and printing • Test Bank software	

AP Calculus AB Unit 3

Unit title: Integration

Unit summary: The definition of a definite integral is provided, and various methods for estimating definite integrals are explored, including Riemann Sums. Skill will be developed both in the interpretation of definite integrals, including area, volume, motion, and accumulation functions, as well as in the use of the properties of integrals to evaluate definite integrals. An understanding of the relationship between integration and differentiation, as expressed in the Second Fundamental Theorem of Calculus, will be developed and will be used to analyze functions defined by integrals.

Primary interdisciplinary connections: Science, Technology, and Engineering

21st Century Themes: Problem Solving, Critical Thinking, Collaboration, Career Skills

Learning Targets

NJSLS Standards: 9.1.12.A.1, 9.1.12.F.2, F.IF.2-3, F.BF.1-2, F.LE.5, A.SSE.1, A.SSE.1b

Content Statements:

1	Antiderivatives and indefinite integration
2	Area
3	Riemann Sums and definite integrals
4	The Fundamental Theorem of Calculus

5 The Mean Value Theorem for Integrals
6 Average value of a function on an interval
7 The Second Fundamental Theorem of Calculus
8 Integration by substitution; change of variables
9 Numerical integration; Trapezoidal Rule
10 Area of a region between two curves
11 Rectilinear motion
12 Volume of a solid with known cross sections

Big Idea: Integrals are used in a wide variety of practical and theoretical applications. They provide the ability to determine areas of shapes with curved edges, volumes of solids with curved surfaces, accumulations of materials governed by dynamic functions over any given interval, and the average values of functions on given intervals.

Unit Essential Questions:

- What is an antiderivative?
- What is the limit definition of an integral of a continuous function on an interval?
- How are definite integrals approximated for functions that are represented

Unit Enduring Understandings:

- Anti-differentiation is the inverse process of differentiation
- The definite integral of a function over an integral is the limit of a Riemann Sum over that integral and can be calculated using a variety of

graphically? numerically? algebraically? verbally? • Can definite integrals be applied to functions with removable or jump discontinuous? • How is the definite integral used to define new functions? • In order to use antiderivatives to solve differential equations, what information is required?	strategies. • The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. • The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. • Anti-differentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.

Unit Learning Targets
Students will…

- Recognize anti-derivatives of basic functions
- Interpret the definite integral as the limit of a Riemann Sum
- Express the limit of a Riemann Sum in integral notation
- Approximate a definite integral
- Calculate a definite integral using areas and properties of definite integrals
- Analyze functions defined by an integral
- Calculate anti-derivatives
- Evaluate definite integrals
- Interpret the meaning of a definite integral within a problem
- Apply definite integrals to problems involving the average value of a function
- Apply definite integrals to problems involving motion
- Apply definite integrals to problems involving area and volume
- Analyze differential equations to obtain general and specific solutions

Evidence of Learning

Summative Assessment: Chapter Tests, AP Practice exam questions

Formative Assessments:

- Independent Practice
- Group practice
- Classroom projects

Lesson Plans

Activities	*Timeframe*
• Explore functions and indefinite integrals: Students have to match cards that show a graphical representation of a function, a verbal description of the function, a graphical representation of some indefinite integrals of the function, and a verbal description of the indefinite integrals, for a variety of functions. The graphical representations of the indefinite integrals, and their descriptions, are then used as examples for the introduction of slope fields.	Weeks 14-20
Teacher Resources	*Teacher Note*
• Textbooks • Classroom set of graphing calculators • Graph paper • Textbook resources available for downloading and printing • AP College Board resources available for downloading and printing • Test Bank software	

AP Calculus AB Unit 4

Unit title: Transcendental Functions and Differential Equations

Unit summary: Understand the calculus definition of the natural logarithmic function. Find derivatives and integrals of the natural logarithmic function, and of exponential functions. Identify inverse functions and find derivatives of inverse functions. Use slope fields to estimate the graphs of, or identify characteristics of functions and graphs. Solve differential equations. Find indefinite solutions to differential equations, and find definite solutions to differential equations, when provide with initial conditions or a point.

Primary interdisciplinary connections: Science, Technology, and Engineering

21st Century Themes: Problem Solving, Critical Thinking, Collaboration, Career Skills

Learning Targets

NJSLS Standards: 9.1.12.A.1, 9.1.12.F.2, F.IF.2, F.IF.4, F.BF.1, F.LE.5

Content Statements:

1	The Natural Logarithmic Function – Differentiation
2	The Natural Logarithmic Function – Integration
3	Inverse Functions
4	Exponential Functions – Differentiation and Integration

Big Idea: The Natural Logarithm is defined as the anti-derivative of 1/x. The derivative of the natural exponential function, e^x, is the same as the function (e^x). Thus these natural functions are an integral part of the concepts of calculus. These transcendental functions also are the model for many natural phenomena on earth.

Unit Essential Questions:

- What is the natural logarithm?
- What are the characteristics of the natural logarithmic function?
- What is the number "e"?
- What information is needed to find a definite solution for a differential equation?
- Can the separation of variables method be used to solve any differential equations?
- What is a slope field?

Unit Enduring Understandings:

- The natural logarithm is defined as the anti-derivative of 1/x.
- The derivative of the natural logarithm of u (ln u) is u'/u.
- The integral of u'/u is ln $|x|$
- The derivative of the natural exponential function, e^u, is $e^u u'$
- Some differential equations can be solved by separation of variables
- Solutions to differential equations may be subject to domain restrictions.

Unit Learning Targets
Students will…

- Develop and use properties of the natural logarithmic function
- Understand the definition of the number "e"
- Find derivatives of functions involving the natural logarithmic function
- Use the log rule for integration to integrate a rational function
- Integrate trigonometric functions
- Verify that one function is the inverse function of another function
- Determine whether a function has an inverse function
- Find the derivative of an inverse function
- Develop properties of the natural exponential function
- Differentiate natural exponential functions
- Integrate natural exponential functions
- Recognize and solve differential equations that can be solved by separation of variables

Evidence of Learning

Summative Assessment: Chapter Tests, AP Practice exam questions

Formative Assessments:

- Independent Practice
- Group practice
- Classroom projects

Lesson Plans

Activities	*Timeframe*
• Explore functions and indefinite integrals: Students will match cards that show a graphical representation of a function, a verbal description of the function, a graphical representation of some indefinite integrals of the function, and a verbal description of the indefinite integrals, for a variety of functions. The graphical representations of the indefinite integrals, and their descriptions, are then used as examples for the introduction of slope fields.	Weeks 21-28
Teacher Resources	*Teacher Note*
• Textbooks	

• Classroom set of graphing calculators • Graph paper • Textbook resources available for downloading and printing • AP College Board resources available for downloading and printing • Test Bank software	

AP Calculus AB Unit 5

Unit title: AP Calculus AB Exam Practice and Review

Unit summary: This unit provides the practice and preparation essential to getting students ready for the AP Calculus AB Exam. Students will have the opportunity to complete ample amounts of practice exercises including AP level multiple choice and open-ended problems. This unit will completely familiarize the students with the format of the test as well as test taking strategies to increase their chance at success on the exam.

Primary interdisciplinary connections: Science, Technology, and Engineering

21st Century Themes: Problem Solving, Critical Thinking, Collaboration, Career Skills

Learning Targets

NJSLS Standards: 9.1.12.A.1, 9.1.12.F.2, F.IF.1-9, F.BF.1-5

Content Statements:

1	Complete multiple choice problems testing proficiency in a wide variety of topics
2	Complete free-response problems demonstrating the ability to solve problems involving a more extended chain of reasoning

Big Idea: AP Calculus AB focuses on conceptual understanding of limits, derivatives, and integrals. Course work encourages students to become logical thinkers, learning to write the solutions to problems in a connected, step-by-step manner with explanatory sentences.

Unit Essential Questions:

- How do we apply our knowledge of calculus to prepare for the AP exam?
- How can we communicate our mathematical thinking clearly?
- How is a graphing calculator used as a problem solving tool?

Unit Enduring Understandings:

- Math can be communicated verbally, analytically, numerically, graphically, and using technology.

Unit Learning Targets
Students will…

- Complete multiple choice problems testing proficiency in a wide variety of topics
- Complete free-response problems demonstrating the ability to solve problems involving a more extended chain of reasoning

Evidence of Learning

Summative Assessment: AP Practice exam questions

Formative Assessments:

- Independent Practice
- Group practice
- Classroom projects

Lesson Plans

Activities	*Timeframe*
Students will solve multiple choice and free response questions at the level of difficulty of The College Board Advanced Placement Calculus AB Examination.	Weeks 28-33
Teacher Resources	*Teacher Note*
• Textbooks • Classroom set of graphing calculators • Graph paper • Textbook resources available for downloading and printing • AP College Board resources available for downloading and printing • Test Bank software	

AP Calculus AB Unit 6

Unit title: AP Calculus AB Final Project

Unit summary: This unit provides the opportunity for students to work with other classmates to collaborate on a year-end final project. Students will have the chance to build on the content and skills attained throughout the course and apply them in a project.

Primary interdisciplinary connections: Science, Technology, and Engineering

21st Century Themes: Problem Solving, Critical Thinking, Collaboration, Career Skills

Learning Targets

NJSLS Standards: 9.1.12.A.1, 9.1.12.F.2, standards will depend on project completed

Content Statements:

1	Complete a year-end final project

Big Idea: AP Calculus AB focuses on conceptual understanding of limits, derivatives, and integrals. Course work encourages students to become logical thinkers, learning to write the solutions to problems in a connected, step-by-step manner with explanatory sentences.

Unit Essential Questions:

- How do we demonstrate proficiency of calculus?
- How can we evaluate, solve and present a complex real world calculus problem?

Unit Enduring Understandings:

- Math can be represented and communicated verbally, analytically, numerically, and visually.

Unit Learning Targets

Students will…

- Complete and collaborate on a year-end final project

Evidence of Learning

Summative Assessment: AP year-end project

Formative Assessments:

- Independent Practice
- Group practice
- Classroom projects

Lesson Plans	
Activities	*Timeframe*
Students will complete a year-end final project collaboratively.	Weeks 34-38
Teacher Resources	*Teacher Note*
• Textbooks • Classroom set of graphing calculators • Graph paper • Textbook resources available for downloading and printing • AP College Board resources available for downloading and printing • Test Bank software • https://sites.google.com/site/sigmaatahsm/home/post-ap-calculus	