Kenilworth Public Schools

Curriculum Guide

Content Area	Mathematics – Calculus
Grade	Twelfth
Revised	May 25, 2017
BOE Approved	10/15/13

Calculus- Grade 12 Scope and Sequence

Unit 1- Pre-calculus and Limits	Unit 2- Derivatives	Unit 3- Applications of Derivatives:	Unit 4- Integrals
Weeks 1-10	Weeks 11-20	Weeks 21-30	Weeks 31-40
Unit Description: Students will be able to perform advanced manipulation and analysis to functions in order to derive limits	*Unit Description*: Students will find instantaneous rates of change at any point within a function or relation	*Unit Description*: Students will use derivatives to further analysis and understanding of real-world phenomena	*Unit Description*: Students will understand the concept of an integral and apply that concept in a variety of ways to a variety of situations
Unit Targets: • Graph and Identify All major function categories • Have deep understanding of Polynomial and trigonometric function behavior • Have deep understanding of and be able to manipulate composite functions • Understand behavior of and be able to graph piece-wise functions	*Unit Targets:* • Define a derivative using different methods • Apply multiple derivative rules on functions and relations • Be able to model related rate problems • Be able to make linear approximations • Define differentials*	*Unit Targets:* • Identify all extrema and inflections • Apply Mean Value Theorem and Rolle's Theorem • Test and analyze concavity • Have deep function analysis applied to curve sketching • Perform modeling and optimizations of real-world phenomena	*Unit Targets:* • Use Riemann Sums and trapezoids to estimate area under functions • Apply anti-derivative rules • Apply the 1st and 2nd Fundamental Theorems of Calculus to solve Integrals • Model totals as a function of integrals • Apply Integration rules and perform u-substitution • Solve differential equations using separation of variables technique*. • Solve growth and decay using integrals*

*AP Calculus topic

Calculus- Grade 12 Unit 1

Unit title: Pre-Calculus and Limits

Unit summary: Students will be able to apply essential Algebra and Pre-calculus techniques to introductory Calculus problems. Students will understand that calculus is the mathematics of change. Students will be able to determine the values a function is approaching even when the exact value does not exist (a limit).

Primary interdisciplinary connections: History, Science, Engineering, Economics, Health, Physical Education

21st Century Themes: Collaboration, Communication, Computer Technology, Creativity, Critical Thinking, Learning Skills, Problem Solving, Technology Skills, Business and Entrepreneurial Literacy

Learning Targets

Standards: NJSLS 9-12.N-RN, 9-12.A-SSE.1-3, 9-12.A-APR.1, 9-12.A-APR.3-7, 9-12.A-CED.1-4, 9-12.A-REI.1-7, 9-12.A-REI.10-11, 9-12.F-IF.1-9, 9-12.F-BF.1, 9-12.F-BF.3-5, 9-12.F-LE.1-5, 9-12.F-TF.1-9

Content Statements:

1	Determine Domain and Range for complex function
2	Graph complex function
3	Determine vertical and horizontal asymptotes
4	Manipulate compound functions
5	Solve limits graphically using technology and via understanding of function behavior
6	Solve limits numerically using technology and by formulating supported conjectures
7	Solve limits analytically using a variety of Algebraic techniques (i.e. Rationalization, Factoring, applying Conjugates, Using Trigonometry)

Big Idea: Calculus is the mathematics of change and allows us to work with both the infinite and the infinitesimal.

Unit Essential Questions:

- What are the properties of linear, quadratic, exponential, parametric, and logarithmic equations?
- What is the difference between average and instantaneous speed?
- What is the connection between one-sided limits and overall limits?
- How can tables be used to determine

Unit Enduring Understandings:

- All problems can be approached graphically, numerically, and analytically
- Mathematics is a continuum where every accepted technique justifies the next, more advanced technique

limits?	

Unit Learning Targets
Students will…

- Graph and identify all major function categories
- Have deep understanding of Polynomial and trigonometric function behavior
- Have deep understanding of and be able to manipulate composite functions
- Understand behavior of and be able to graph piece-wise functions
- Calculate a variety of limits

Evidence of Learning

Summative Assessment:

- Quizzes (Primarily open-ended free response format)
- Tests (Primarily standardized test format)

Formative Assessments:

- Presentation of techniques on chalk board, SmartBoard, and via document camera
- Homework review
- Class discussion
- Self Evaluation (self scoring open-ended problems according to a College Board style rubric)

Lesson Plans

Activities	*Timeframe*
• Graph as many categories of functions as possible Work collaboratively as a class Each student presents a unique type of function and elaborates on its characteristics	20-30 minutes during first week of class
Teacher Resources	*Teacher Note*
• Textbook • SmartBoard • Document Camera • College Board website • Khan Academy	Students can present on SmartBoard, chalkboard, or document cameras with communicators

• Various online graphing utilities and calculus calculators • Graphing software (MS Mathematics, Winplot, etc.)	

Calculus- Grade 12 Unit 2

Unit title: Derivatives

Unit summary: Students will be able to identify, define and calculate derivatives, or instantaneous rates of change, in a variety of problems.

Primary interdisciplinary connections: History, Science, Engineering, Economics, Health, Physical Education

21st Century Themes: Collaboration, Communication, Computer Technology, Creativity, Critical Thinking, Learning Skills, Problem Solving, Technology Skills, Business and Entrepreneurial Literacy

Learning Targets

Standards: NJSLS 9-12.N-RN, 9-12.A-SSE.1-3, 9-12.A-APR.1, 9-12.A-APR.3-7, 9-12.A-CED.1-4, 9-12.A-REI.1-7, 9-12.A-REI.10-11, 9-12.F-IF.1-9, 9-12.F-BF.1, 9-12.F-BF.3-5, 9-12.F-LE.1-5, 9-12.F-TF.1-9

Content Statements:

1	Define and test for continuity
2	Apply limit definition of a derivative
3	Define and manipulate derivative as an instantaneous slope
4	Memorize and apply derivative formulas
5	Define and test for differentiability
6	Use derivatives to define the relationship between position, velocity, and acceleration
7	Use derivatives to calculate related rates in real world problems
8	Implement implicit differentiation for non-functions

Big Idea: A derivative is a rate of change. The real world is full of constantly changing values and derivatives model the world far more realistically than can be done with algebra alone.

Unit Essential Questions:

- What is the difference between continuity at a point and a continuous function?
- How does continuity play a role in the Intermediate Value Theorem?
- What is the relationship between slope of a tangent line and its normal line?
- What are the properties needed to graph a derivative f' from the original function f?
- How are one-sided derivatives related to a function's overall derivative being

Unit Enduring Understandings:

- Most theorems in calculus only apply if we can first establish continuity and differentiability
- Understanding the relationship between a function and its derivative is the first step to truly understanding how to model situations that change

defined? • What are the cases where f' fails to exist? Why? • What is the relationship between differentiability and continuity? Is the relationship reversible? • What is the connection between finding a derivative by definition and finding a derivative using integer power rules? • How do velocity, speed, and acceleration relate to each other in terms of derivatives? • How can the properties of a function's and its derivative's graphs connect to velocity, speed, and acceleration? • What determines if a chain rule is needed to find a function's derivative? • What determines how many chains are needed when using the power chain rule? • When is implicit differentiation needed? • How can tangents' and normal lines'slopes be calculated from an implicitly defined function?	

Unit Learning Targets
Students will…

- Define a derivative using different methods
- Apply multiple derivative rules on functions and relations
- Model velocity and acceleration
- Be able to model related rate problems
- Be able to make linear approximations
- Define differentials
-

Evidence of Learning

Summative Assessment:

- Quizzes (Primarily open-ended free response format)
- Tests (Primarily standardized test format)

Formative Assessments:

- Presentation of techniques on chalk board, SmartBoard, and via document camera
- Homework review
- Class discussion
- Self-Evaluation (self-scoring open-ended problems according to a College Board style rubric)

Lesson Plans

Activities	*Timeframe*
• Working in groups, students will visually estimate slopes for a variety of different graphs • Prior to knowing derivative formulas, students will use algebraic knowledge to calculate as accurate a slope as possible • Groups will present their findings, and most precise group will be recognized	20-30 minutes Beginning of Unit 2
Teacher Resources	*Teacher Note*
• Textbook • SmartBoard • Document Camera • College Board website • Khan Academy • Various online graphing utilities and calculus calculators • Graphing software (MS Mathematics, Winplot, etc.)	Graphs may be presented on SmartBoard, passed out in a handout, or created by student groups

Calculus- Grade 12 Unit 3

Unit title: Applications of Derivatives

Unit summary: Students will use derivatives for further analysis and understanding of real-world phenomena.

Primary interdisciplinary connections: History, Science, Engineering, Economics, Health, Physical Education

21st Century Themes: Collaboration, Communication, Computer Technology, Creativity, Critical Thinking, Learning Skills, Problem Solving, Technology Skills, Business and Entrepreneurial Literacy

Learning Targets

Standards: NJSLS 9-12.N-RN, 9-12.A-SSE.1-3, 9-12.A-APR.1, 9-12.A-APR.3-7, 9-12.A-CED.2, 9-12.A-REI.1-7, 9-12.A-REI.10-11, 9-12.F-IF.1-9, 9-12.F-BF.1, 9-12.F-BF.3-5, 9-12.F-LE.1-5, 9-12.F-TF.1-9

Content Statements:

1	Find extrema
2	Develop deep understanding of Rolle's Theorem and Mean Value Theorem
3	Implement the First Derivative Test analyzing increasing and decreasing functions
4	Use concavity and the Second Derivative Test
5	Apply First and Second Derivatives to curve sketching
6	Develop and implement optimization procedures

Big Idea: Derivatives can determine when extreme values occur (most profit, least materials, best practice, etc.) and are used to model optimization scenarios.

Unit Essential Questions:

- How are extreme values calculated by hand? On a calculator?
- What is the connection between critical points and extrema?
- How are max/min and intervals of increasing/decreasing represented on a graph?
- How is the mean value theorem related to continuity and differentiability?
- What is the connection between critical points, max/min, and points of reflection?
- How are max/min intervals of

Unit Enduring Understandings:

- When analyzing functions, finding the extremes is often the most important piece of analysis
- Derivatives are a valuable tool in optimizing many types of processes
- Inflection points are descriptive in analysis of statistics and economics

increasing/decreasing, and concavity represented on a graph? • How can properties of a function be deduced from its first and second derivatives? • How can that function then be represented on a graph? • How can derivatives be applied to maximizing profits and minimizing costs? • How can these values be found on a graph?	

Unit Learning Targets
Students will…

- Identify all extrema and inflections
- Apply Mean Value Theorem and Rolle's Theorem
- Test and analyze concavity
- Have deep function analysis applied to curve sketching
- Perform modeling and optimizations of real-world phenomena

Evidence of Learning

Summative Assessment:

- Quizzes (Primarily open-ended free response format)
- Tests (Primarily standardized test format)

Formative Assessments:

- Presentation of techniques on chalk board, SmartBoard, and via document camera
- Homework review
- Class discussion
- Self-Evaluation (self-scoring open-ended problems according to a College Board style rubric)

Lesson Plans

Activities	*Timeframe*
• Using a flat piece of paper, students will fold up the sides to produce a container with the maximum volume. Measure each student's volume. Make conjectures regarding properties and measurements of optimized	20-30 minutes Beginning of Unit 3

volume. Use differential calculus to compare experimental results with ideal optimized results.	
Teacher Resources	*Teacher Note*
• Textbook • SmartBoard • Document Camera • College Board website • Khan Academy • Various online graphing utilities and calculus calculators • Graphing software (MS Mathematics, Winplot, etc.)	Based on available time, construction may begin either in class or as a homework assignment

Calculus- Grade 12 Unit 4

Unit title: Integration

Unit summary: Students will understand the concept of an integral and apply that concept in a variety of ways to a variety of situations

Primary interdisciplinary connections: History, Science, Engineering, Economics, Health, Physical Education

21st Century Themes: Collaboration, Communication, Computer Technology, Creativity, Critical Thinking, Learning Skills, Problem Solving, Technology Skills, Business and Entrepreneurial Literacy

Learning Targets

Standards: NJSLS 9-12.N-RN, 9-12.A-SSE.1-3, 9-12.A-APR.1, 9-12.A-APR.3-7, 9-12.A-CED.2, 9-12.A-REI.1-7, 9-12.A-REI.10-11, 9-12.F-IF.1-9, 9-12.F-BF.1, 9-12.F-BF.3-5, 9-12.F-LE.1-5, 9-12.F-TF.1-9

Content Statements:

1	Solve antiderivatives and indefinite integration
2	Calculate area under complex curves
3	Calculate Reimann Sums and definite integrals
4	Understand and apply the Fundamental Theorem of Calculus
5	Apply integration by substitution
6	Calculate numerical integration
7	Work with natural logarithms and natural exponents
8	Explore growth and decay
9	Find area between curves

Big Idea: While derivatives analyzed "rates of change," integrals can find cumulative totals.

Unit Essential Questions:

- How accurately does the rectangular approximation method calculate the area under a curve?
- How are the approximations for area under the curve and volume of a sphere similar?
- How are Riemann sums used in the rectangular approximation method and approximations for volume of a sphere?
- How is each term in the integral notation connected to Riemann sums?

Unit Enduring Understandings:

- Simpler models can be extended to more detailed and accurate models
- Problems can be solved using graphical, numerical, and analytical techniques

• How can a graphing calculator be used to calculate integrals/area under a curve? • How can these be calculated on a graphing calculator? • What does average value of an integral find?	

Unit Learning Targets
Students will…

- Use Riemann Sums and trapezoids to estimate area under functions
- Apply anti-derivative rules
- Apply the 1st and 2nd Fundamental Theorems of Calculus to solve integrals
- Model totals as a function of integrals
- Apply integration rules and perform u-substitution
- Solve differential equations using separation of variables technique*
- Solve growth and decay using integrals*

Evidence of Learning

Summative Assessment:

- Quizzes (Primarily open-ended free response format)
- Tests (Primarily standardized test format)

Formative Assessments:

- Presentation of techniques on chalk board, SmartBoard, and via document camera
- Homework review
- Class discussion
- Self-Evaluation (self-scoring open-ended problems according to a College Board style rubric)

Lesson Plans

Activities	*Timeframe*
• Students will cut out rectangles of various dimensions. Students will "fit" rectangles on a graph of a smooth curve and attempt to approximate the total area. Class will discuss other basic geometric shapes that might be used to better approximate the area under the curve.	20-30 minutes (first week of Unit 4)
• Students will use online Riemann Sum Calculators to approximate integrals using different amounts of	20 minutes (second week of Unit 4)

rectangles.	
Teacher Resources	*Teacher Note*
• Textbook • SmartBoard • Document Camera • College Board website • Khan Academy • Various online graphing utilities and calculus calculators • Graphing software (MS Mathematics, Winplot, etc.) • Cardboard, scissors, graphing paper	Pre-AP Geometry Workbook has a similar activity that can be referenced